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THE TRIPLEABLENESS THEOREM

JONATHAN MOCK BECK

Preface by Robert Paré

The 1960's was an exciting time for category theory, the beginning of the Lawvere Era. The scene was dominated by a dozen or so larger-than-life personalities: Lawvere, Lambek, and Isbell to name just three. There was much activity, the results coming quickly, often not written up for publication. The people in the know knew.

Jon Beck was a prominent contributor to this group and was notoriously lax when it came to publication, continually honing and refining his results. He relied on conference talks, face to face discussions and the occasional hand-written notes, Xeroxed, passed around and Xeroxed again.

His thesis (reproduced in TAC Reprints) set the stage for the rapid development of monad theory, with an intense flurry of activity during the "Zurich Triples Year" (1965-66).

Monads (then called triples) were originally designed as a tool for generating cohomology theories and the relation to universal algebra was just beginning to emerge. In order to understand what these cohomologies were calculating, at least in low degree, Beck needed a condition he called "tripleability", now monadicity. It then became desirable to get useful conditions insuring monadicity, and this is what he did. The "crude tripleability theorem" or "CTT" gave easy-to-check conditions insuring tripleability. PTT, the precise tripleability theorem, was only a bit harder to check and gave necessary and sufficient conditions. It is amazing that such simple conditions as the existence and preservation of certain coequalizers, together with adjointness of course, are all that is needed. Especially since this characterizes categories of universal algebras. This he wrote up in the beautiful untitled and undated manuscript now being reproduced here in the TAC reprint series.

In the years since, monads have proven to be a central concept in category theory and the monadicity theorems powerful and invaluable tools, and appear in most of the standard textbooks on categories. The publication now of Beck's original manuscript provides an important historical document.

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Editors' note

This reprint would not exist without the work of Nathanael Arkor, who first put in an enormous amount of effort—ultimately successful—to track down a copy of this widely cited but hard to find manuscript, then raised the idea of publishing it as a TAC Reprint, then typed it up in LaTeX. We are very grateful to him for this work.

The manuscript was originally distributed at a conference held at the Seattle Research Center of the Battelle Memorial Insitute in 1968, and the copy used here was provided by John Kennison, to whom we are also grateful. We also thank Nadine Beck for giving permission for the publication of this reprint, and to Michael Barr for contacting her.

Some notation and terminology is now outmoded, but we have left it untouched. The most obvious instances are 'triples' and 'tripleableness' for what we now call monads and monadicity. Beck also used the diagrammatic order of composition that was common among category theorists then, including $X\mu$ to mean the X-component of a natural transformation μ, now usually written as μ_X.

The only significant change is that a small number of references have been added (and again, this is the work of Nathanael Arkor). There were none in the original manuscript.

1 Introduction

Let $\mathbf{A} \underset{U}{\overset{F}{\rightleftarrows}} \mathbf{B}$ with $F \dashv U$. Write $\eta\colon 1_{\mathbf{A}} \to FU$, $\epsilon\colon UF \to 1_{\mathbf{B}}$ for the unit and counit of the adjointness. Then $\mathbb{T} = (T, \eta, \mu)$ is a triple in $\mathbf{A}$, where $T = FU$, $\eta\colon 1_{\mathbf{A}} \to T$, $\mu = F\epsilon U\colon T^2 \to T$. We have the category of $\mathbb{T}$-algebras $\mathbf{A}^{\mathbb{T}}$ as defined by Eilenberg–Moore [Eilenberg and Moore (1965)], $F^{\mathbb{T}}\colon \mathbf{A} \to \mathbf{A}^{\mathbb{T}}$ by $X \rightsquigarrow (XT, X\mu)$, $U^{\mathbb{T}}\colon \mathbf{A}^{\mathbb{T}} \to \mathbf{A}$ by $(X, \xi) \rightsquigarrow X$, and $F^{\mathbb{T}} \dashv U^{\mathbb{T}}$.

$$\begin{array}{ccc} \mathbf{A}^{\mathbb{T}} & \xleftarrow{\quad\phi\quad} & \mathbf{B} \\ & {\scriptstyle U^{\mathbb{T}}}\searrow \quad \swarrow{\scriptstyle U} & \\ & \mathbf{A} & \end{array}$$

is defined by $Y\phi = (YU, Y\epsilon U)$. The adjoint pair $F \dashv U$ is *tripleable* if $\phi^{\vee} \dashv \phi$ exists such that the unit and counit are isomorphisms $1_{\mathbf{A}^{\mathbb{T}}} \xrightarrow{\sim} \phi^{\vee}\phi$, $\phi\phi^{\vee} \xrightarrow{\sim} 1_{\mathbf{B}}$. Given U, this property is independent of which left adjoint F is used, so we also say U *is tripleable* in this situation. It seems to be too much to ask for $\phi^{\vee}\phi = \mathbf{A}^{\mathbb{T}}$, $\phi\phi^{\vee} = \mathbf{B}$. On the other hand, in category theory, the usual "equivalences" of categories should be replaced by adjoint equivalences.

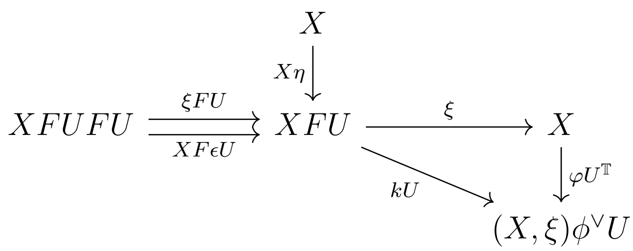
2 Crude tripleableness theorem

2.1 Theorem *If $\mathbf{B}$ has coequalizers and U preserves and reflects coequalizers, then U is tripleable. (It is assumed $F \dashv U$ exists.)*

PROOF $\phi^\vee$ is the coequalizer: $XFUF \overset{\xi F}{\underset{XF\epsilon}{\rightrightarrows}} XF \xrightarrow{k} (X,\xi)\phi^\vee$. One way of proving this is by verifying the sequence of set isomorphisms[1]

$$\begin{aligned} &\text{maps } (X,\xi) \xrightarrow{f} Y\phi \\ \to\ &\text{maps } X \xrightarrow{f} YU \text{ such that } \xi f = fFU \,.\, Y\epsilon U \\ \to\ &\text{maps } XF \xrightarrow{g} Y \text{ such that } \xi F \,.\, g = XF\epsilon \,.\, g \\ \to\ &\text{maps } (X,\xi)\phi^\vee \xrightarrow{g} Y. \end{aligned}$$

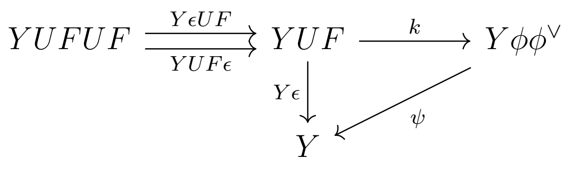
If $(X,\xi) \xrightarrow{\varphi} (X,\xi)\phi^\vee\phi$ denotes the unit of $\phi^\vee \dashv \phi$, then $\varphi U^{\mathbb{T}} = X\eta \,.\, kU$.



Now, $\xi = \text{coeq}(\xi FU, XF\epsilon U)$ for if some $XFU \xrightarrow{z} Z$ coequalizes ξFU and $XF\epsilon U$, then $X \xrightarrow{X\eta . z} Z$ is the unique map such…[2] But $kU = \text{coeq}(\xi FU, XF\epsilon U)$ since U preserves coequalizers. Moreover,

$$\xi(\varphi U^{\mathbb{T}}) = \xi \,.\, X\eta \,.\, kU = XFU\eta \,.\, \xi FU \,.\, kU = XFU\eta \,.\, XF\epsilon U \,.\, kU = kU.$$

Therefore $\varphi U^{\mathbb{T}}$ is an isomorphism, and since $U^{\mathbb{T}}$ reflects isomorphisms, so is φ. The counit $Y\phi\phi^\vee \xrightarrow{\psi} Y$ is defined by its appearance in the diagram below.



We proved above that the $\mathbb{T}$-structure of an algebra is a coequalizer, so if U is applied to $(Y\epsilon UF, YUF\epsilon, Y\epsilon)$ we get a coequalizer diagram in $\mathbf{A}$ ($Y\epsilon U$ is the $\mathbb{T}$-structure of the algebra $Y\phi$). But U reflects coequalizers, so $Y\epsilon = \text{coeq}(Y\epsilon UF, YUF\epsilon)$. Therefore ψ is an isomorph. ■

[1] Editors' note: in the original manuscript, the third line of the display that follows ends with 'such that $\xi F = XF\epsilon \,.\, g$', but this is surely an error.

[2] Editors' note: here and on two later occasions, a sentence in the manuscript trails off with an ellipsis.

3 Contractible coequalizers

A diagram $Y_1 \underset{d_1}{\overset{d_0}{\rightrightarrows}} Y_0 \xrightarrow{d} Y$ with $d_0 d = d_1 d$ looks like the 1-skeleton of an augmented simplicial object. (Here degeneracies will be ignored.) A *contraction* of a simplicial object is a sequence of maps $h_n\colon Y_n \to Y_{n+1}$ such that $h_n d_i = d_i h_{n-1}$ for $0 \le i \le n$ and $h_n d_{n+1} = Y_n$. (You can also use $h_n d_0 = Y_n$, $h_n d_i = d_{i-1} h_n$.) We are led to look at diagrams

$$Y_1 \overset{h_0}{\curvearrowleft} \underset{d_1}{\overset{d_0}{\rightrightarrows}} Y_0 \overset{h_{-1}}{\curvearrowleft} \xrightarrow[d]{} Y$$

such that $d_0 d = d_1 d$, $h_{-1} d = Y$, $h_0 d_0 = d h_{-1}$, $h_0 d_1 = Y_0$. In this case $d = \text{coeq}(d_0, d_1)$, for if $d_0 z = d_1 z$ for $Y_0 \xrightarrow{z} Z$ then $h_{-1} z\colon Y \to Z$ is the unique map such… Thus we call such a diagram a *contractible coequalizer diagram.*

If $\mathbf{A} \xleftarrow{U} \mathbf{B}$, we call coequalizer data $Y_1 \underset{d_1}{\overset{d_0}{\rightrightarrows}} Y_0$ *U-contractible* if there are Z, d, h_{-1}, h_0 in $\mathbf{A}$ such that

$$Y_1 U \overset{h_0}{\curvearrowleft} \underset{d_1 U}{\overset{d_0 U}{\rightrightarrows}} Y_0 U \overset{h_{-1}}{\curvearrowleft} \xrightarrow[d]{} Z$$

is a contractible coequalizer diagram. We say: $\mathbf{B}$ *has U-contractible coequalizers* if all U-contractible coequalizer data in $\mathbf{B}$ have coequalizers in $\mathbf{B}$; *U preserves U-contractible coequalizers* if whenever $Y_1 \rightrightarrows Y_0$ is U-contractible and has a coequalizer $Y_0 \to Y$ in $\mathbf{B}$, then the canonical map $Z \to YU$ is an isomorphism; *U reflects U-contractible coequalizers* if $Y_1 \rightrightarrows Y_0 \to Y$ being mapped into a contractible coequalizer diagram by U implies that $Y_1 \rightrightarrows Y_0 \to Y$ is a coequalizer diagram in $\mathbf{B}$.

$$\left[\; Y_1 U \overset{h_0}{\curvearrowleft} \underset{d_1 U}{\overset{d_0 U}{\rightrightarrows}} Y_0 U \overset{h_{-1}}{\curvearrowleft} \xrightarrow[dU]{} YU \;\right]$$

($Y_1 \rightrightarrows Y_0 \to Y$ will not necessarily be contractible in $\mathbf{B}$.)

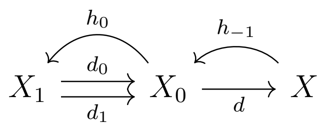
4 Precise tripleableness theorem

4.1 Theorem *U is tripleable $\iff$ $\mathbf{B}$ has, and U preserves and reflects, U-contractible coequalizers.*

Proof $\Leftarrow$ is clear. One only has to notice that all coequalizers arising in the proof of the crude theorem were U-contractible.

$\Rightarrow$: We can assume $\mathbf{B} = \mathbf{A}^{\mathbb{T}}$ and prove that $\mathbf{A}^{\mathbb{T}}$ has $U^{\mathbb{T}}$-contractible coequalizers. (The (dual) example of comodules over a non-flat coalgebra shows that $\mathbf{A}^{\mathbb{T}}$ need not have all coequalizers. But it follows from a result of Linton's alluded to below that $\mathbf{A}^{\mathbb{T}}$ has all coequalizers if $\mathbf{A} =$ sets.)

Let $(X_1, \xi_1) \underset{d_1}{\overset{d_0}{\rightrightarrows}} (X_0, \xi_0)$ be $U^{\mathbb{T}}$-contractible, i.e. we have the accompanying diagram in $\mathbf{A}$.



Let $XT \xrightarrow{\xi} X$ be $h_{-1}T \,.\, \xi_0 d$. Then $dT \,.\, \xi = \xi_0 d$.

$$\begin{array}{ccc} X_0T & \xrightarrow{dT} & XT \\ {\scriptstyle \xi_0}\downarrow & & \downarrow{\scriptstyle \xi} \\ X_0 & \xrightarrow[d]{} & X \end{array}$$

For

$$\begin{aligned} dT \,.\, \xi &= dT \,.\, h_{-1}T \,.\, \xi_0 d = (dh_{-1})T \,.\, \xi_0 d = (h_0 d_0)T \,.\, \xi_0 d = h_0 T \,.\, d_0 T \,.\, \xi_0 d \\ &= h_0 T \,.\, \xi_1 d_0 d = h_0 T \,.\, \xi_1 d_1 d = h_0 T \,.\, d_1 T \,.\, \xi_0 d = (h_0 d_1)T \,.\, \xi_0 d = \xi_0 d. \end{aligned}$$

This shows that $d\colon X_0 \to X$ is compatible with $\mathbb{T}$-structures. Since $h_{-1}d = X$, it follows that (X, ξ) is a $\mathbb{T}$-algebra. Also if a different contraction h'_0, h'_{-1} were used, and ξ' defined as $h'_{-1}T \,.\, \xi_0 d$, then $\xi' = \xi$, since $\xi = (h_{-1}d)T \,.\, \xi = h_{-1}T \,.\, dT \,.\, \xi = h_{-1}T \,.\, \xi_0 d$, and $\xi' = (h_{-1}d)T = h_{-1}T \,.\, dT \,.\, \xi' = h_{-1}T \,.\, \xi_0 d$ also. Thus the $\mathbb{T}$-structure ξ is well-defined. Finally, $d = \mathrm{coeq}(d_0, d_1)$, for if $(X_0, \xi_0) \xrightarrow{y} (Y, \theta)$ coequalizes d_0, d_1, then $(X, \xi) \xrightarrow{h_{-1}y} (Y, \theta)$ is the unique...[3] The above construction shows that $U^{\mathbb{T}}$ preserves and reflects $U^{\mathbb{T}}$-contractible coequalizers. ∎

5 Remarks

It should be possible to improve the above theorem (apart from streamlining the exposition). Conditions implying tripleableness should be found which are easier to verify in practice. For instance, the following is true:

$$U \text{ is tripleable} \iff \begin{array}{l} \mathbf{B} \text{ has and } U \text{ preserves } U\text{-contractible coequalizers,} \\ \text{and } U \text{ reflects isomorphisms.} \end{array}$$

It seems to follow without much difficulty, from this, that algebraic or varietal categories are tripleable over sets (and Linton can prove tripleable categories are varietal [Linton (1966), Linton (1969)]).

[3] Note that h_{-1} is not an algebra map, but $h_{-1}y$ is.

References

[Eilenberg and Moore (1965)] Samuel Eilenberg and John C. Moore. Adjoint functors and triples. *Illinois Journal of Mathematics*, 9(3):381–398, 1965.

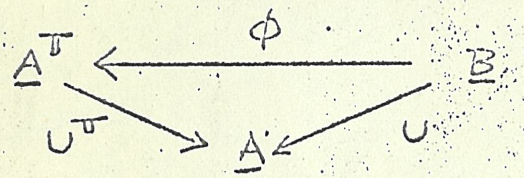
[Linton (1966)] Fred E. J. Linton. Some aspects of equational categories. In *Proceedings of the Conference on Categorical Algebra*, pages 84–94. Springer, 1966.

[Linton (1969)] Fred E. J. Linton. An outline of functorial semantics. In *Seminar on triples and categorical homology theory*, pages 7–52. Springer, 1969.

~~The Tripleableness theorem~~ by Jon Beck

"UNTITLED MANUSCRIPT".

1\. Let $\underline{A} \underset{U}{\overset{F}{\rightleftarrows}} \underline{B}$ with $F \dashv U$. Write $\eta : 1_{\underline{A}} \to FU$, $\epsilon : UF \to 1_{\underline{B}}$ for the unit and counit of the adjointness. Then $\mathbb{T} = (T, \eta, \mu)$ is a triple in $\underline{A}$, where $T = FU$, $\eta : 1_{\underline{A}} \to T$, $\mu = F\epsilon U : T^2 \to T$. We have the category of $\mathbb{T}$-algebras $\underline{A}^{\mathbb{T}}$ as defined by Eilenberg-Moore, $F^{\mathbb{T}} : \underline{A} \to \underline{A}^{\mathbb{T}}$ by $X \rightsquigarrow (XT, X\mu)$, $U^{\mathbb{T}} : \underline{A}^{\mathbb{T}} \to \underline{A}$ by $(X, \xi) \rightsquigarrow X$, and $F^{\mathbb{T}} \dashv U^{\mathbb{T}}$.



is defined by $Y\Phi = (YU, Y\epsilon U)$. The adjoint pair $F \dashv U$ is <u>tripleable</u> if $\Phi^{\vee} \dashv \Phi$ exists such that the unit and counit are isomorphisms: $1_{\underline{A}^{\mathbb{T}}} \xrightarrow{\sim} \Phi^{\vee}\Phi$, $\Phi\Phi^{\vee} \xrightarrow{\sim} 1_{\underline{B}}$. Given U, this property is independent of which left adjoint F is used, so we also say, <u>U is tripleable</u> in this situation. It seems to be too much to ask for $\Phi^{\vee}\Phi = \underline{A}^{\mathbb{T}}$, $\Phi\Phi^{\vee} = \underline{B}$. On the other hand, in category theory, the usual "equivalences" of categories should be replaced by adjoint equivalences.

2\. <u>Crude tripleableness theorem.</u> If $\underline{B}$ has coequalizers and U preserves and reflects coequalizers, then U is tripleable. (It is assumed $F \dashv U$ exists.)

<u>Proof</u>. $\Phi^{\vee}$ is the coequalizer: $XFUF \underset{XF\epsilon}{\overset{\xi F}{\rightrightarrows}} XF \xrightarrow{k} (X,\xi)\Phi^{\vee}$. One way of proving this is by verifying the sequence of set isomorphisms

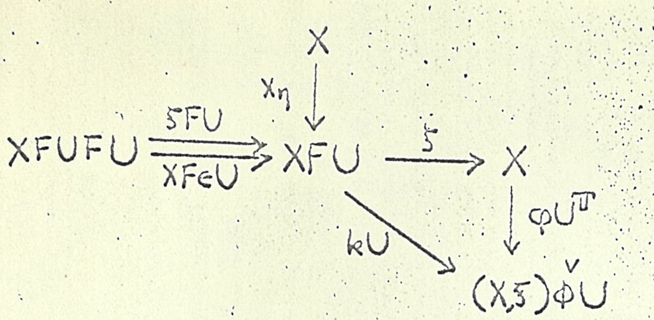
maps $(X,\xi) \xrightarrow{f} Y\Phi$

$\longrightarrow$ maps $X \xrightarrow{f} YU$ such that $\xi f = \xi FU . Y\epsilon U$

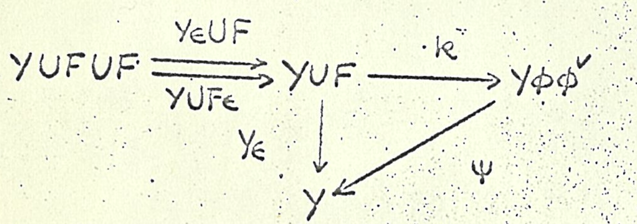
$\longrightarrow$ maps $XF \xrightarrow{g} Y$ such that $\xi F = XF\epsilon . g$

$\longrightarrow$ maps $(X,\xi)\Phi^{\vee} \xrightarrow{g} Y$.

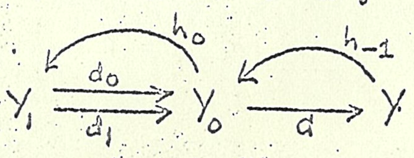
If $(X,\xi) \xrightarrow{\varphi} (X,\xi)\phi^\vee\phi$ denotes the unit of $\phi^\vee \dashv \phi$, then $\varphi U^\pi = X\eta . kU$



Now, $\xi = \text{coeq}(\xi FU, XF\epsilon U)$ for if some $XFU \xrightarrow{z} Z$ coequalizes ξFU and $XF\epsilon U$, then $X \xrightarrow{X\eta . z} Z$ is the unique map such . . . But: $kU = \text{coeq}(\xi FU, XF\epsilon U)$ since U preserves coequalizers. Moreover, $\xi(\varphi U^\pi) = \xi . X\eta . kU = XFU\eta . \xi FU . kU = XFU\eta . XF\epsilon U . kU = kU$. Therefore φU^π is an isomorphism, and since U^π reflects isomorphism, so is φ. The counit $Y\phi\phi^\vee \xrightarrow{\psi} Y$ is defined by its appearance in the diagram below. We proved above that the π-structure of an algebra is a coequalizer, so if U is applied to $(Y\epsilon UF, YUF\epsilon, Y\epsilon)$ we get a coequalizer diagram in $\underline{A}$ ($Y\epsilon U$ is the π-structure of the algebra $Y\phi$). But U reflects coequalizers, so, $Y\epsilon = \text{coeq}(Y\epsilon UF, YUF\epsilon)$. Therefore ψ is an isomorph.



3. Contractible coequalizers. A diagram $Y_1 \overset{d_0}{\underset{d_1}{\rightrightarrows}} Y_0 \xrightarrow{d} Y$ with $d_0 d = d_1 d$ looks like the 1-skeleton of an augmented simplicial object. (Here degeneracies will be ignored.) A contraction of a simplicial objects is a sequence of maps $h_n : Y_n \longrightarrow Y_{n+1}$ such that $h_n d_i = d_i h_{n-1}$ for $0 \le i \le n$ and $h_n d_{n+1} = Y_n$. (You can also use $h_n d_0 = Y_n$, $h_n d_i = d_{i-1} h_n$.) We are led to look at diagrams such that $d_0 d = d_1 d$, $h_{-1} d = Y$, $h_0 d_0 = d h_{-1}$, $h_0 d_1 = Y_0$. In this case $d = \text{coeq}(d_0, d_1)$, for if $d_0 z = d_1 z$ for $Y_0 \xrightarrow{z} Z$ then $h_{-1} z : Y \longrightarrow Z$ is the unique map such ... Thus we call such a diagram a contractible coequalizer diagram.



If $\underline{A} \xleftarrow{U} \underline{B}$, we call coequalizer data $Y_1 \overset{d_0}{\underset{d_1}{\rightrightarrows}} Y_0$ <u>U-contractible</u> if there are Z, d, h_{-1}, h_0 in $\underline{A}$ such that

$$Y_1U \overset{d_0U}{\underset{d_1U}{\rightrightarrows}} Y_0U \xrightarrow{d} Z \qquad (h_0 : Y_0U \to Y_1U,\ h_{-1} : Z \to Y_0U)$$

is a contractible coequalizer diagram.

We say: $\underline{B}$ <u>has</u> <u>U-contractible coequalizers</u> if all U-contractible coequalizer data in $\underline{B}$ have coequalizers in $\underline{B}$; U <u>preserves U-contractible coequalizers</u> if whenever $Y_1 \rightrightarrows Y_0$ is U-contractible and has a coequalizer $Y_0 \to Y$ in $\underline{B}$, then the canonical map $Z \to YU$ is an isomorphism; U <u>reflects</u> <u>U-contractible coequalizers</u> if $Y_1 \rightrightarrows Y_0 \to Y$ being mapped into a contractible coequalizer diagram by U implies that $Y_1 \rightrightarrows Y_0 \to Y$ is a coequalizer diagram in $\underline{B}$.

$$\left[\ Y_1U \overset{d_0U}{\underset{d_1U}{\rightrightarrows}} Y_0U \xrightarrow{dU} YU \qquad (h_0,\ h_{-1})\ \right]$$

($Y_1 \rightrightarrows Y_0 \to Y$ will not necessarily be contractible in $\underline{B}$.)

4. <u>Precise tripleableness theorem.</u> U is tripleable $\iff$ $\underline{B}$ has, and U preserves and reflects, U-contractible coequalizers.

<u>Proof</u>. $\Leftarrow$ is clear. One only has to notice that all coequalizers arising in the proof of the crude theorem were U-contractible.

$\Rightarrow$: We can assume $\underline{B} = \underline{A}^{\mathbf{T}}$ and prove that $\underline{A}^{\mathbf{T}}$ has $U^{\mathbf{T}}$-contractible coequalizers. (The (dual) example of comodules over a non-flat coalgebra shows that $\underline{A}^{\mathbf{T}}$ need not have all coequalizers. But it follows from a result of Linton's alluded to below that $\underline{A}^{\mathbf{T}}$ has all coequalizers if $\underline{A} =$ sets.) Let $(X_1, \xi_1) \overset{d_0}{\underset{d_1}{\rightrightarrows}} (X_0, \xi_0)$ be $U^{\mathbf{T}}$-contractible, i.e. we have the accompanying diagram in $\underline{A}$. Let $XT \xrightarrow{\xi} X$ be $h_{-1}T . \xi_0 d$. Then $dT . \xi = \xi_0 d$. For

$$X_1 \overset{d_0}{\underset{d_1}{\rightrightarrows}} X_0 \xrightarrow{d} X \qquad (h_0,\ h_{-1})$$

$$\begin{array}{ccc} X_0T & \xrightarrow{dT} & XT \\ \downarrow{\scriptstyle \xi_0} & & \downarrow{\scriptstyle \xi} \\ X_0 & \xrightarrow{d} & X \end{array}$$

$$dT . \xi = dT . h_{-1}T . \xi_0 d = (dh_{-1})T . \xi_0 d =$$

$= (h_0 d_0)T \cdot \xi_0 d = h_0 T \cdot d_0 T \cdot \xi_0 d = h_0 T \cdot \xi_1 d_0 d = h_0 T \cdot \xi_1 d_1 d$
$= h_0 T \cdot d_1 T \cdot \xi_0 d = (h_0 d_1) T \cdot \xi_0 d = \xi_0 d.$

This shows that $d : X_0 \longrightarrow X$ is compatible with $\mathbb{T}$-structures. Since $h_{-1} d = X$, it follows that (X, ξ) is a $\mathbb{T}$-algebra. Also if a different contraction h_0', h_{-1}' were used, and ξ' defined as $h_{-1}' T \cdot \xi_0 d$, then $\xi' = \xi$, since $\xi = (h_{-1} d) T \cdot \xi = h_{-1} T \cdot dT \cdot \xi = h_{-1} T \cdot \xi_0 d$, and $\xi' = (h_{-1} d) T \cdot$ $= h_{-1} T \cdot dT \cdot \xi' = h_{-1} T \cdot \xi_0 d$ also. Thus the $\mathbb{T}$-structure ξ is well-defined. Finally, $d = \mathrm{coeq}(d_0, d_1)$ for if $(X_0, \xi_0) \xrightarrow{y} (Y, \theta)$ coequalizes d_0, d_1, then $(X, \xi) \xrightarrow{h_{-1} y} (Y, \Theta)$ is the unique.. *) The above construction shows that $U^{\mathbb{T}}$ preserves and reflects $U^{\mathbb{T}}$-contractible coequalizers.

*) Note that h_{-1} is not an algebra map, but $h_{-1} y$ is.

5. Remarks. It should be possible to improve the above theorem (apart from streamlining the exposition). Conditions implying tripleableness should be found which are easier to verify in practice. For instance, the following is true:

U is tripleable $\Longleftrightarrow$ $\underline{B}$ has and U preserves U-contractible coequalizers, and U reflects isomorphisms.

It seems to follow without much difficulty, from this, that algebraic or varietal categories are tripleable / $\mathcal{S}$ (and Linton can prove tripleable categories are varietal).

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